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$$(A_1 + A_5)(A_3 - A_2) = (-A_1 - A_3 + A_2 + A_4 - A_3 + A_6 + A_2 - A_5) \\ = -\{1 + (A_3 - A_2)\}.$$

$$\text{Similarly, } (A_1 + A_5)(-A_4 - A_6) = -\{1 + (A_1 + A_5)\}; (A_3 - A_2) \\ (-A_4 - A_6) = -\{1 + (-A_4 - A_6)\}.$$

$$\therefore \{ (A_1 + A_5)(A_3 - A_2) + (A_1 + A_5)(-A_4 - A_6) + (A_3 - A_2) \\ (-A_4 - A_6) \} = -4.$$

$$\text{Now } (A_1 + A_5)(A_3 - A_2)(-A_4 - A_6) = -(-A_4 - A_6) - (A_3 - A_2) \\ (-A_4 - A_6) = A_4 + A_6 + 1 + (-A_4 - A_6) = 1.$$

Hence,  $(A_1 + A_5)$ ,  $(A_3 - A_2)$ ,  $(-A_4 - A_6)$  are the three roots of the cubic  $x^3 - x^2 - 4x - 1 = 0$ . Call them  $A$ ,  $B$ ,  $C$ , respectively.

$$\therefore A_1 + A_5 = A; A_1 \cdot A_5 = (A_4 + A_6) = -C.$$

$$A_3 - A_2 = B; -A_3 \cdot A_2 = -(A_1 + A_5) = -A.$$

$$-A_4 - A_6 = C; A_4 \cdot A_6 = (A_2 - A_3) = -B.$$

Hence,  $A_1$  and  $A_5$  are the roots of  $x^2 - Ax - C = 0$ .

$A_3$  and  $-A_2$  are the roots of  $x^2 - Bx - A = 0$ .

$-A_4$  and  $-A_6$  are the roots of  $x^2 - Cx - B = 0$ .

## NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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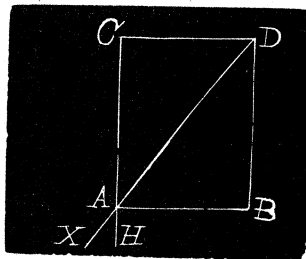
[Continued from the September Number.]

*Given any triangle (fig. 7)  $ABD$  right angled at  $B$ ; prolong  $DA$  at any point  $X$ , and through  $A$  erect  $HAC$  perpendicular to  $AB$ .*

*I say the external angle  $XAH$  will be equal, or less, or greater than the internal and opposite  $ADB$ , according as is true the hypothesis of right angle, or obtuse angle, or acute angle: and inversely.*

**Proof.** Assume in  $HC$  the portion  $AC$  equal to  $BD$ , and join  $CD$ .  $CD$  will be, in the hypothesis of right angle (P. III.) equal to  $AB$ . Wherefore the angle  $ADB$  will be equal (Eu. I. 8.) to the angle  $DAC$ , or to its equal (Eu. I. 15.) to the angle  $XAH$ . Quod erat primo loco demonstrandum.

Then, in the hypothesis of obtuse angle,  $CD$  will be (P. III.) less than  $AB$ .



Wherefore in the triangle  $ADB$ , the angle  $DAC$ , or its vertical  $XAH$ , will be (Eu. I. 25.) less than the angle  $ADB$ . Quod erat secundo loco demonstrandum.

While; in the hypothesis of acute angle,  $CD$  will be (P. III.) greater than the opposite  $AB$ . Wherefore in the said triangle the angle  $DAC$ , or its verticle  $XAH$ , will be (Eu. I. 25.) greater than the angle  $ADA$ . Quod erat tertio loco demonstrandum.

But inversely: if the angle  $CAD$ , or its vertical  $XAH$ , be equal to the internal and opposite  $ADB$ ; the join  $CD$  will be (Eu. I. 4.) equal to  $AB$ , and therefore the hypothesis of right angle will be (P. IV.) true.

But if however the angle  $CAD$ , or its vertical  $XAH$ , be less, or greater than the internal or opposite  $ADB$ ; also the join  $CD$  will be (Eu. I. 24.) less or greater than  $AB$ ; and therefore (P. IV.) will be true respectively the hypothesis of obtuse angle, or acute angle. Quod omnia erant demonstranda.

*Proposition IX. In any right-angled triangle the two acute angles remaining are taken together equal to one right angle, in the hypothesis of right angle; greater than one right angle, in the hypothesis of obtuse angle; but less in the hypothesis of acute angle.*

**Proof.** For if the angle  $XAH$  (fig. 7.) is equal to the angle  $ADB$ , which is certain, from the preceding proposition, in the hypothesis of right angle, then the angle  $ADB$  makes up with the angle  $HAD$  two right angles, as (Eu. I. 13.) the aforesaid angle  $XAH$  makes them up with this angle  $HAB$  being subtracted, the two angles  $ADB$  and  $BAD$  remain together equal to one right angle. Quod erat primum.

However, if the angle  $XAH$  is less than the angle  $ADB$ , which is certain from the preceding proposition, in the hypothesis of obtuse angle, then the angle  $ADB$  makes up with the angle  $HAD$  more than two right angles, since with it (Eu. I. 13.) the angle  $XAH$  makes up two. Wherefore, the angle  $HAB$  being subtracted, the two angles  $ADB$  and  $BAD$  will be together greater than one right angle. Quod erat secundum.

Finally, if the angle  $XAH$  be greater than the angle  $ADB$ , which is certain from the preceding proposition in the hypothesis of acute angle, then the angle  $ADB$  will make up less than two right angles with the angle  $HAD$ , since with this (Eu. I. 13.) the angle  $XAH$  makes up two. Wherefore, subtracting the right angle  $HAB$ , the angles  $ADB$  and  $BAD$  will be together less than one right angle. Quod erat tertium.

